AP Calculus AB

Integration Review

1) $\int_1^2 (2x-3x^2+4x^3)dx$

$\left[x^2-x^3+x^4+C\right]\Big|_1^2$

$[4-8+16]-[1-1+1]$

*2) $\int_0^1 x^6(4x^2+x-5)dx$

$\int_0^1 (4x^8+x^7-5x^6)\,dx$

$\left[\frac{4}{9}x^9+\frac{1}{8}x^8-\frac{5}{7}x^7+C\right]\Big|_0^1$

$\left[\frac{4}{9}+\frac{1}{8}-\frac{5}{7}\right]-[0]$

3) $\int_3^4 3x(x^2-8)^{-1}dx$

$\left[\frac{3}{2}\ln(x^2-8)+C\right]\Big|_3^4$

$\left[\frac{3}{2}\ln 8\right]-\left[\frac{3}{2}\ln 1\right]$

$\boxed{\frac{3}{2}\ln 8}$

check

$\frac{2x}{x^2-8}$

$2(\ )=3$

$(\ )=\frac{3}{2}$

4) $\int 7e^{-5x}dx$

$\boxed{-\frac{7}{5}e^{-5x}+C}$

*5) $\int \cos^5 x \sin x$

$\boxed{-\frac{1}{6}\cos^6 x+C}$

check

$6\cos^5 x(-\sin x)$

6) $\int_{\ln 2}^{\ln 9} 4e^x dx$

$\left[4e^x+C\right]\Big|_{\ln 2}^{\ln 9}$

$4e^{\ln 9}-4e^{\ln 2}$

$36 - 8 = \boxed{28}$

*7) $\int_4^{25} \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int_4^{25} x^{-1/2}e^{x^{1/2}}dx$

$\left[2e^{x^{1/2}}+C\right]\Big|_4^{25}$

check

$e^{x^{1/2}}\cdot\frac{1}{2}x^{-1/2}$

$\boxed{2e^5-2e^2}$

8) $\int_{\sqrt{3}/3}^{1} \frac{dx}{1+x^2}$

$\left[\arctan x+C\right]_{\sqrt{3}/3}^{1}$

$[\arctan 1]-[\arctan \sqrt{3}/3]$

$\boxed{\frac{\pi}{4}-\frac{\pi}{6}}$

9) $\int \cos(3-2x)\,dx$

$\boxed{-\frac{1}{2}\sin(3-2x)+C}$

check

$\cos(3-2x)\cdot(-2)$

10) $\int x(9-x^2)^{-1/2}\,dx$

$\boxed{-(9-x^2)^{1/2}+C}$

check:

$\frac{1}{\cancel{2}}(9-x^2)^{-1/2}(\cancel{-2}x)$

11) $\int 3x(x^2+1)^{-4}\,dx$

$\boxed{-\frac{1}{2}(x^2+1)^{-3}+C}$

check

$-3(x^2+1)^{-4}\cdot 2x$

* 12) $\int x\cos x^2\,dx$

$\boxed{\frac{1}{2}\sin x^2+C}$

check

$\cos x^2\cdot\ 2x$

* 13) $\int \cos^3 2u\,\sin 2u\,du$

$\int (\cos 2u)^3\sin 2u\,du$

$\boxed{-\frac{1}{8}(\cos 2u)^4+C}$

check:

$4(\cos 2u)^3\cdot(-\sin 2u)(2)$

14) $\int \frac{2\cos x}{\sin^2 x}\,dx = \int 2\cos x\,(\sin x)^{-2}dx$

$\boxed{2(\sin x)^{-1}+C}$

check:

$-1(\sin x)^{-2}\cdot\cos x$

* 15) $f'(x)=\frac{1}{2}x^2+\frac{3}{4}x$

$f(x)=\frac{1}{6}x^3+\frac{3}{8}x^2+C$

$2=\frac{1}{6}+\frac{3}{8}+C$

$35/24=C$

$f(x)=\frac{1}{6}x^3+\frac{3}{8}x^2+35/24$

16) $f'(x)=5x^4-2x$

$f(x)=x^5-x^2+C$

$6=1-1+C$

$6=C$

$f(x)=x^5-x^2+6$

$f(2)=32-4+6$

$\boxed{f(2)=34}$

17) $\lim_{x\to 4}\frac{x^2-4}{x^2-16}\to\frac{12}{0}$

DNE

18) $\lim_{x\to 6}\frac{x-6}{x^2-36}\to\frac{0}{0}$

Using L'Hopital's

$\lim_{x\to 6}\frac{1}{2x}=\boxed{\frac{1}{12}}$

19) $f''(x)=x$

$f'(x)=\frac{1}{2}x^2+C$

$0=2+C$

$-2=C$

$f'(x)=\frac{1}{2}x^2-2$

$f(x)=\frac{1}{6}x^3-2x+C$

$4=0-0+C$

$4=C$

$f(x)=\frac{1}{6}x^3-2x+4$

$f(1)=\frac{1}{6}-2+4$

20) $\int_0^\pi \sec x\tan x\,dx$

$\sec x\big|_0^\pi$

$(\sec\pi)-(\sec 0)$

$\boxed{-2}$

* 21) $\int_0^1 e^x(3-e^x)^{-2}dx$

$\left[(3-e^x)^{-1}+C\right]\Big|_0^1$

$(3-e)^{-1}-\frac{1}{2}$

$\frac{1}{3-e}-\frac{1}{2}$